# Formation and transport of a sand heap in an inclined and vertically vibrated container

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We investigate experimentally the formation and the transportation of a heap formed by granular materials in an inclined and vertically vibrated container. We observe how the transport velocity of heap up the container is related to the driving acceleration, the driving frequency, and the inclination of the container. An empirical law which governs the transport velocity of the heap is presented. An analogous experiment was performed with a heap-shaped Plexiglas block. We propose that the compressive force resulted from pressure gradient in ambient gas plays a crucial role in enhancing and maintaining a heap, and the ratchet effect causes the movement of the heap.

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It is well known that the granular materials turn out to play a number of unusual behaviors such as segregation, convection, heaping, density wave, and anomalous sound propagation, etc. [1]. However, due to the complexities, the properties of such materials are not well understood. For example, heaping is one long-standing problem since Faraday [2–5]. Several physical mechanisms have been identified as possible causes of heaping: friction between the walls and particles [6], analog of acoustic streaming if the shaking is nonuniform [7], gas pressure effect [8], and autoamplification [9]. Recently we reported briefly the experimental observation of formation and transportation of heap formed by granular materials in an inclined and vertically vibrated container [10]. In this paper, we will give more systematic experimental result, and especially stress the mechanism of formation and transportation of the heap.

The experimental setup is the same as that in Ref. [10]. We investigated the behavior of two types of quartz sands: spheres of diameter 0.15–0.20 mm and grains of irregular shape or coarse surface with diameter 0.3–0.5 mm. The container was inclined with an inclination $\alpha$ changed from 0 rad (horizontal) to 0.25 rad by putting a pad underneath it. The vibration exciter (Brüel & Kjær 4805) was driven by a sinusoidal signal, and controlled by a vibration exciter control (Brüel & Kjær 1050). Driving frequency $f$ and dimensionless acceleration amplitude $\Gamma=4\pi^2 f^2 A_0/g$ (where $A_0$ is the driving amplitude and $g$ the gravitational acceleration) were used as two control parameters. The experiment shows that even if in horizontal container the horizontal acceleration, as long as with nonzero component in longer direction of the container, can cause movement of heap along longer direction of the container. The horizontal acceleration of our exciter is about 2.5% of the vertical acceleration. To get rid of the influence of the horizontal component of the driving acceleration on the movement of the heap along the longer direction of the container, we adjusted the container so that its longer direction is perpendicular to horizontal acceleration of the exciter. The ranges of $\Gamma$ and $f$ we used were from 1.4 to 2.8 and from 11 Hz to 20 Hz, respectively, which are good ranges for heap formation. At first, about 80 ml of sands was uniformly put into the lower part of the vessel, and the depth of the layer is about 50 in diameter for coarse grains and 100 for spherical grains. Then as $\Gamma$ increased to and beyond some critical acceleration $\Gamma_c$ (changes from 1.2 to 1.3 as frequency increases from 11 Hz to 20 Hz), center-high heaps formed, meanwhile they moved up the container. As center-high heaps reached higher end wall of the container, they moved forward still until they became wall-high heaps. Figure 1 shows the photos of heaps formed by coarse sands. Figure 1(a) is center-high heap, and (b) is wall-high heap. For center-high heap, the back (or right shown as in the figure) surface is longer than the frontal (or left) one, but the dynamical angles of repose (slightly smaller than the maximum angle of repose of the static heap) of both frontal and back surfaces relative to horizontal are the same. The difference between the heaps formed by two types of sands is the dynamical angle of repose of the heap formed by coarse sands (0.38 rad) is larger than that formed by spherical sands (0.33 rad). The heaps moved up the container with nearly uniform velocity. We measured the velocities $V$ of the heaps. Figure 2 gives the results for coarse sands. Figure 2(a) shows $V$ vs $\Gamma$ for four different frequencies. One can see that $V$ increases with $\Gamma$ for all of the frequencies, but decreases as $f$ increases for all values of $\Gamma$. The velocity of the heap formed by coarse sands is greater than that of the heap formed by spherical sands for all sets of the values of $\Gamma$ and $f$. Figure 2(c) shows the velocity of heap as a function of inclination of the container. It is shown that $V$ increases as $\alpha$, and behaves in the same way as in Fig. 2(a) as $f$ changes. Similarly, the heap formed by coarse sands moves faster than the heap formed by spherical sands does for all sets of the values of $\alpha$ and $f$. The experimental data in Figs. 2(a) and 2(c) are well fitted by the equation

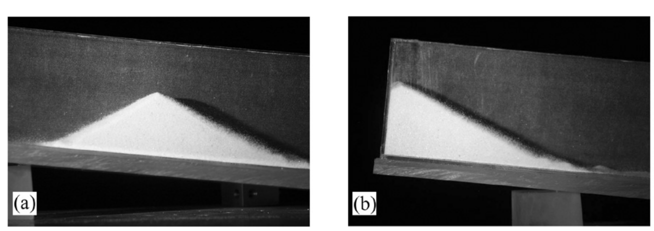


FIG. 1. The center-high heap (a) and the wall-high heap (b) of coarse sands. $\alpha=0.087$ rad, $f=15$ Hz, $\Gamma=2$.

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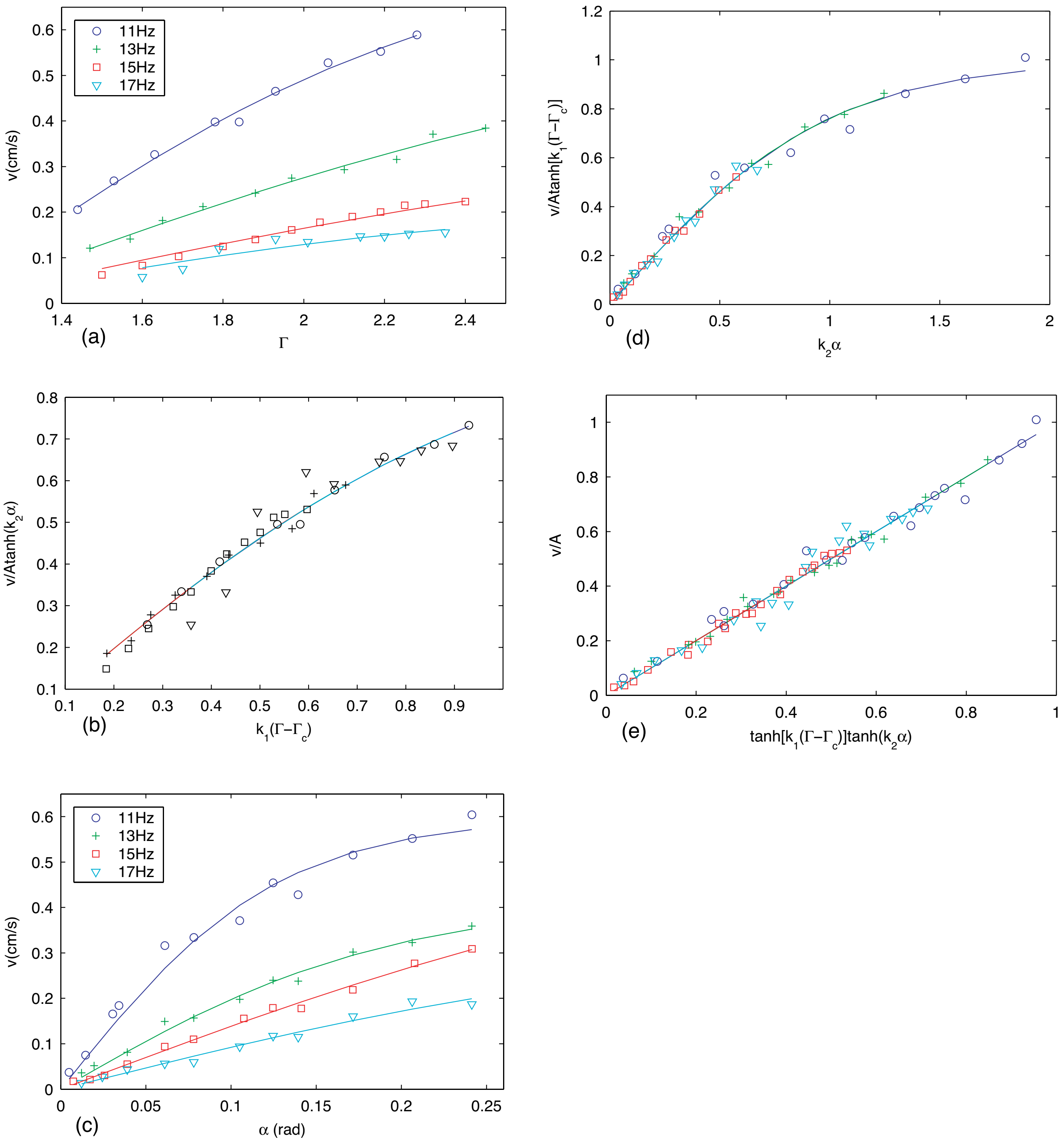


FIG. 2. (Color online) The transport velocity $V$ of heap formed by coarse sands. (a) $V$ vs $\Gamma$ for different frequencies (the data are adapted from Ref. [10]). The solid lines are fits by $V=A\tanh[k_1(\Gamma-\Gamma_c)]\tanh(k_2\alpha)$ with $\alpha=0.045$ rad. (b) $V/A\tanh(k_2\alpha)$ vs $k_1(\Gamma-\Gamma_c)$ for the data in (a). The solid line is $\tanh[k_1(\Gamma-\Gamma_c)]$. (c) $V$ vs $\alpha$ for different frequencies. The solid lines are fits by $V=A\tanh[k_1(\Gamma-\Gamma_c)]\tanh(k_2\alpha)$ with $\Gamma=2$. (d) $V/A\tanh[k_1(\Gamma-\Gamma_c)]$ vs $k_2\alpha$ for the data in (c). The solid line is $\tanh(k_2\alpha)$. (e) $V/A$ vs $\tanh[k_1(\Gamma-\Gamma_c)]\tanh(k_2\alpha)$ for the data both in (a) and (c). The solid line is $\tanh[k_1(\Gamma-\Gamma_c)]\tanh(k_2\alpha)$. For $f=11$, 13, 15, and 17 Hz, and $\Gamma_c=1.2$, 1.23, 1.26, and 1.28, the fitting parameters are $A=1.16$, 1.04, 1.48, and 0.6 cm/s, $k_1=1.19$, 0.93, 0.77, and 1.32, and $k_2=8.09$, 5.17, 2.38, and 2.78 rad$^{-1}$, respectively.

$$V=A\tanh[k_1(\Gamma-\Gamma_c)]\tanh(k_2\alpha), \qquad (1)$$

with $\alpha=0.045$ rad for Fig. 2(a) and $\Gamma=2$ for Fig. 2(c), respectively. The solid lines in the figure are nonlinear least-square fits by Eq. (1). The fitting parameters $A$, $k_1$ and $k_2$ depend on the driving frequency, the properties (e.g., density, size, and shape, etc.) of the grains, and the viscosity coefficient of the interstitial gas. If the data of Fig. 2(a) are plotted

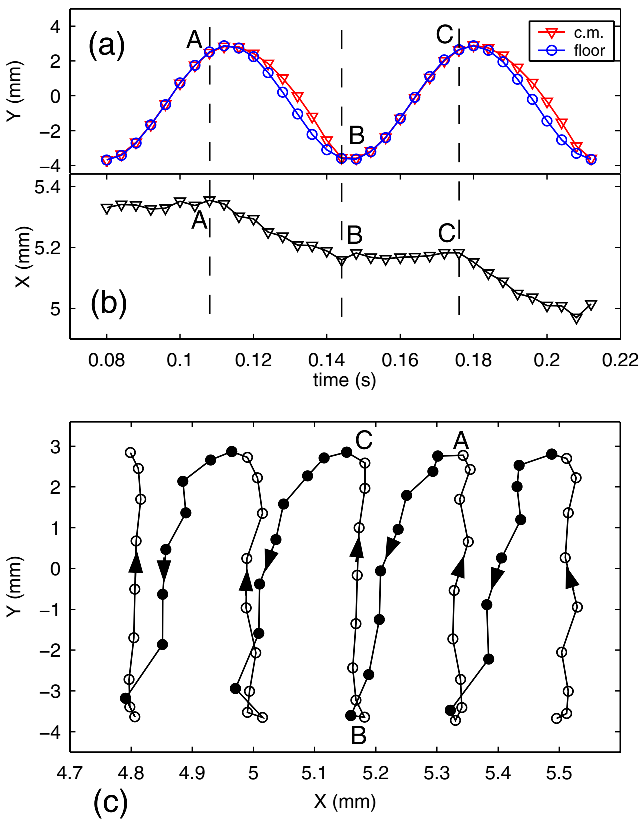


FIG. 3. (Color online) (a) The vertical ($y$) components of the movement of the center of mass of the block (c.m.) and the container (floor) as a function of time. (b) The horizontal ($x$) component of the movement of the center of mass of the block as a function of time. (c) The orbit of the center of mass of the block. The arrows represent the direction of the orbit. The parameters are $\alpha$=0.06 rad, $f$=15 Hz, $\Gamma$=2.3.

as $V/A\ \tanh(k_2\alpha)$ vs $k_1(\Gamma-\Gamma_c)$, they collapse onto a single curve, Fig. 2(b); and if the data of Fig. 2(c) are plotted as $V/A\ \tanh[k_1(\Gamma-\Gamma_c)]$ vs $k_2\alpha$, they also collapse onto a single curve, Fig. 2(d). Moreover, if the data of both Fig. 2(a) and Fig. 2(c) are plotted as $V/A$ vs $\tanh[k_1(\Gamma-\Gamma_c)]\tanh(k_2\alpha)$, they all collapse onto a single curve, Fig. 2(e), within the experimental resolution and for the ranges of our experimental parameters. For $\Gamma$ larger than the upper limit value (which is different for different frequency) in Fig. 2(a), the heap becomes unstable, and the standing wave appears. Because the angle of repose of the granular layer relative to horizontal does not change no matter how the inclination angle of the container is, therefore, to form a heap in an inclined container the inclination of the container should be less than the angle of repose of the granular layer. In fact, for $\alpha > 0.25$ rad in Fig. 2(c), heap cannot well form.

Obviously, the transport of the heap is a cooperative behavior of the granular materials. To examine this idea, we put a Plexiglas block the same in shape and dimension as the sand heap in the same vibrated inclined container, a similar transport of the block up the container was observed. So we consider that the transport of the heap is similar to that of a solid block. But why does it move, or what is the mechanism of the transport? We used a high speed camera (Redlake MASD MotionScope PCI 2000 SC) to record the movement of the block as it moved up the container with record rate of 250 fps (frames per second), then played back slowly (25 fps). In this way, we can see the detail of the movement of the block. We marked the center of mass of the block with a black point, $C$ (shown as in Fig. 4). Fig. 3 shows the movements of the center of mass of the block and the container recorded experimentally in laboratory reference frame. Figure 3(a) shows the vertical ($y$) components of the movement of the center of mass of the block and the container as a function of time. We have shifted the center of mass a little bit for the purpose of better illustration. One can see that from $A$ to $B$ the block separates with container and flies freely, meanwhile in Fig. 3(b) it moves a distance in horizontal ($x$) direction (and up the container). Correspondingly, from $A$ to $B$ in Fig. 3(c), the solid circles represent the part of the vibration cycle when the block flies freely. From $B$ to $C$ in Fig. 3(a), block moves together with container in vertical direction while in Fig. 3(b) it does not move in horizontal direction. This means that block does not slide down (or move up) the container during its collision with container. Correspondingly, from $B$ to $C$ in Fig. 3(c), the open circles represent the part when the block is on the container floor. To examine the effect of ambient gas, we performed two things. First, to get rid of the pressure difference below and above the block, we drilled a number of holes vertically and parallel each other through the block, and observed no transport of the block. Second, we evacuated the air from the container and also observed no transport of the block (either with or without holes drilled through the block). These imply that it is an air pressure difference and not just the presence of air, which is responsible for the movement of the block up the container. Therefore we propose a mechanism for the movement of the block up the container as follows. In each cycle, when $\Gamma\cos 2\pi ft < -1$, block separates with container, and a gap forms between block bottom and the container floor (Fig. 4). The pressure $p_1$ in gap is less than the atmospheric pressure $p_0$ above the block on an average. (Ref. [11] shows that the mean pressure in a gap between the bottom of granular bed and the container floor is below atmospheric pressure. And as Faraday said [12] in this gap, "it forms a partial vacuum." We believe that this also suits to the case of the block.) This pressure difference causes a force exerting on

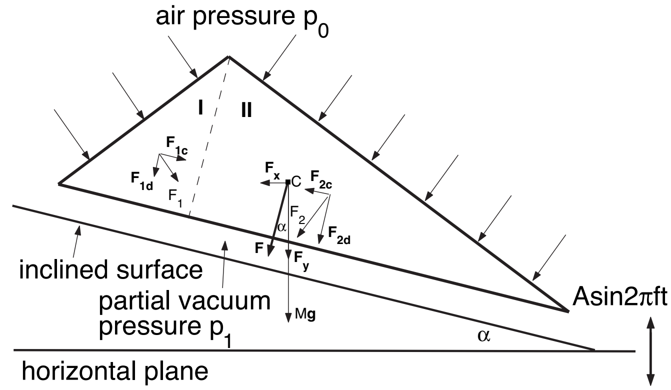


FIG. 4. A schematic diagram for the forces acting on block (heap) during free flight.

the block. As shown in Fig. 4, the forces acting on the frontal (I) and back (II) parts (with dashed line as intersection) are represented by vectors $\mathbf{F_1}$ and $\mathbf{F_2}$ at the centers of mass of two parts, $c_1$ and $c_2$, respectively. They are perpendicular to frontal and back surface (i.e., the frontal and back sides in the figure), respectively. The magnitudes of them are proportional to the lengths of the frontal and back sides, respectively. They are decomposed into two components: those parallel (represented by $\mathbf{F_{1c}}$ and $\mathbf{F_{2c}}$) and those perpendicular (represented by $\mathbf{F_{1d}}$ and $\mathbf{F_{2d}}$) to the bottom of the block, respectively. The parallel components $\mathbf{F_{1c}}$ and $\mathbf{F_{2c}}$ are equal in magnitude but opposite in direction. So the total force $\mathbf{F}$ is only the sum of perpendicular components $\mathbf{F_{1d}}$ and $\mathbf{F_{2d}}$, acts at the center of mass of the block, $C$, is perpendicular to the bottom of block, and makes an angle $\alpha$ (i.e., the inclination of the container) with gravitational force of the block, $M\mathbf{g}$ ($M$ is the mass of the block or heap). This force, with a nonzero horizontal component $\mathbf{F_x}$, together with gravitation force of the block, force the block to move along a ballistic trajectory until it collides with the container [$A \rightarrow B$ in Fig. 3(c)]. Upon colliding with the container, the block moves together with the container in vertical direction until next separation with container [$B \rightarrow C$ in Fig. 3(c)]. Then a new cycle begins. The friction angle between the Plexiglas block and the container floor (i.e., the inclination angle at which the block slides down the container with no shaking) is about 0.27 rad, which is much larger than the angle used in Fig. 3, so the movement of the block sliding down the container is neglected, just as seen in Fig. 3. The analysis above shows that the transport of the block up the container is a ratchet effect caused by the pressure difference above and below the block and the friction force between the block and the container floor. One can see from the analysis above that the shape of the solid block is not important, as long as the force resulted from the pressure difference above and below the block has nonzero horizontal component. In fact, we used other shaped block, e.g., the rectangle, and also observed transportation up the container.

For the granular bed, if for any reason, some small (or flatter) initial heap has formed. In the period of free flight, the pressure difference above and below the heap [11] leads to a pressure gradient (or force) in the interior of the heap. Also cf. Fig. 4, and we still use $\mathbf{F_1}$ and $\mathbf{F_2}$ as representatives of the total force acting on frontal and back parts of the heap, respectively. These two forces are perpendicular to the frontal and back surface, respectively. The couple of parallel components $\mathbf{F_{1c}}$ and $\mathbf{F_{2c}}$ enhance the heap: they compress the heap in the direction parallel to heap bottom, meanwhile the heap is elongated in direction perpendicular to heap bottom. This makes the heap bottom convex, as we have observed experimentally in both inclined [10] and horizontal [9] containers. Figure 5(a) is a schematic diagram of the velocity (or mass flow) field in the interior of the heap. The total force $\mathbf{F}$ acting on the center of mass of the heap, together with gravitational force $M\mathbf{g}$, force the heap as a whole to move along a ballistic trajectory and up the container. Upon colliding with the container, the center part of the heap bottom touches floor first, then the other parts, from center to outer, touch floor consecutively. This results in a further enhancement of the heap. In this way the slope of the heap is getting larger and

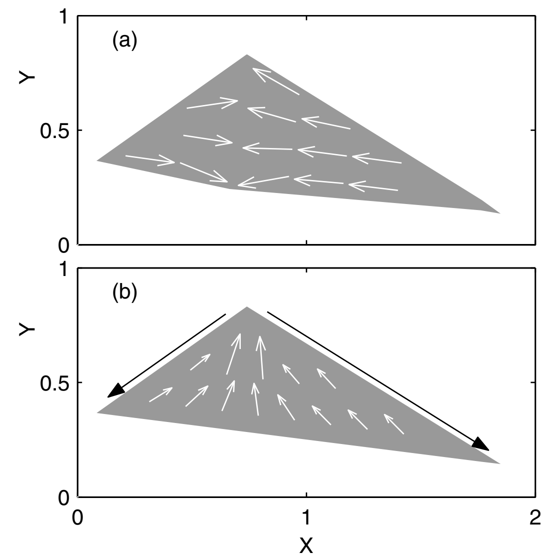


FIG. 5. The schematic diagram of the velocity (or mass flow) field in a heap formed by coarse sands during the period of free flight (a) and heap-floor collision (b), respectively.

larger. Once the slope angle reaches and exceeds the dynamic angle of repose of the heap, the avalanche occurs on the surface of the heap. Then the process repeats periodically: during the free flight, the compressive force forces the slope of the heap exceeding the dynamic angle of repose, and during the bed-floor collision, avalanche occurs. So when the heap reaches a steady state, in the laboratory reference frame, one can see a steady convection flow: in the interior of the heap, grains move upward, while at the surface grains move rapidly downward [schematically shown as in Fig. 5(b)]. This analysis is also suitable to wall-high heap, in which the compressive force points to the higher end wall. Similar to the block, the transport of the heap up the container is also a ratchet effect, which is caused by the pressure difference above and below the heap and the friction force between the heap and the container floor. Here we have also neglected the movement of the heap sliding down the container, because the friction angle of the granular bed is about 0.5 rad, which is much larger than the angles used in Fig. 2(c). If we pump the air out of the container, as pressure is reduced, heap will not well form and reach a flat state gradually. This shows that ambient gas also plays an important role in enhancing and maintaining a heap. Because both the free flight time and the pressure gradient or force in the interior of the heap depend on the driving acceleration and the frequency, the inclination of the container, the properties of grains, and the viscosity of the interstitial gas in a specific and complex way, and change with time and space [13] (the investigation on this problem is under way), we only use the empirical law [Eq. (1)] to describe the transport velocity of the heap at present.

This model is also suitable for the heap (either the center-high or the wall-high heap) formed in a horizontal container, where the compressive force in the ambient gas (as above) plays the same role (enhancing and maintaining the heap) as in an inclined container. The total force $\mathbf{F}$ due to the pressure gradient is perpendicular to heap bottom, i.e., is parallel to gravitational force $M\mathbf{g}$, and no force forces heap to move in the horizontal direction.

In conclusion, the pressure gradient in ambient gas plays a crucial role in enhancing and maintaining a heap in vibrating granular materials; the pressure difference above and below the heap and the friction force between the granular bed and the container floor, which lead to a ratchet effect, are a unique cause for the transport of a heap up an inclined container. The transport velocity of the heap is well described by Eq. (1). Our mechanism for enhancing and maintaining heap also shows that one reason for lacking of heaping in molecular dynamics simulation may be that those models did not take into account the compressive force due to ambient gas effect.

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